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Time evolution of the number-operational phase entangled state in a number-phase entangled Jaynes–Cummings model

Hong-yi Fan[1,2] **and Hai-liang Lu**[2]

[1] CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China
[2] Department of Physics, Shanghai Jiao Tong University, Shanghai 200030, People's Republic of China

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Abstract

We apply the newly derived number-operational phase entangled state to solve a number-phase entangled Jaynes–Cummings model. The time evolution of the phase and number difference is calculated. The model also exhibits some collapse-revival phenomena.

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1. Introduction

In a recent paper [1] we have constructed a new number difference and operational phase entangled state by operating the Noh–Fougères–Mandel (NFM) phase operator [2–4] on the two-mode twin-photon state

$$\|\mathcal{N}_d, m\rangle = (\mathrm{e}^{\mathrm{i}\Theta})^{\mathcal{N}_d}|m, m\rangle \tag{1}$$

where $\mathcal{N}_d$ is an integer, $|m, m\rangle = a^{\dagger m} b^{\dagger m}/m!|0, 0\rangle$,

$$\mathrm{e}^{\mathrm{i}\Theta} = \sqrt{\frac{a^\dagger - b}{a - b^\dagger}} \tag{2}$$

is the NFM phase operator, $[a, a^\dagger] = 1, [b, b^\dagger] = 1$. When $\mathcal{N}_d \geqslant 0$, the Schmidt decomposition of $\|\mathcal{N}_d, m\rangle$ is

$$\|\mathcal{N}_d, m\rangle = \Gamma\left(\frac{\mathcal{N}_d}{2} + 1\right) \sum_{n'=0}^{\infty} \sum_{k=0}^{\min(n',m)} \sqrt{\frac{n'!}{(n' + \mathcal{N}_d)}} \times \binom{-\frac{\mathcal{N}_d}{2}}{n' - k} \binom{\frac{\mathcal{N}_d}{2}}{m - k} \binom{\frac{\mathcal{N}_d}{2} + k}{k} |n' + \mathcal{N}_d\rangle_1 |n'\rangle_2 \tag{3}$$

from which we see that the difference of the photon number in two modes is $\mathcal{N}_d$. Hence the NFM phase operator is essentially an entangling operator. In this paper we shall present an application of the state $\|\mathcal{N}_d, m\rangle$. We shall point out that it plays an essential role in solving a new number-operational phase entangled Jaynes–Cummings model (JCM). Among various interaction models which can describe atom–photon coupling, the JCM [5] is of fundamental importance. Based on it many generalized JCM were proposed [6, 7]. In this work we consider such a generalized model that the atom–photon coupling depends on the net photon-number difference in a cavity into which a nonlinear medium with two-level atoms and two light beams are injected, i.e. that the new model's Hamiltonian is

$$\mathcal{H} = \omega(a^\dagger a - b^\dagger b) + \frac{1}{2}\Omega\sigma_z + \lambda\left[\sigma_+\sqrt{\frac{a-b^\dagger}{a^\dagger - b}}\sqrt{a^\dagger a - b^\dagger b} + \sqrt{a^\dagger a - b^\dagger b}\sqrt{\frac{a^\dagger - b}{a-b^\dagger}}\sigma_-\right] \tag{4}$$

where ω is the frequency of the optical field, Ω is the atomic transition frequency and λ is the coupling constant between the field and the atom. The two-level atomic system is represented by the Pauli spin operators,

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5}$$

obeying

$$[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm \qquad [\sigma_+, \sigma_-] = \sigma_z \tag{6}$$

the term $\sigma_+\sqrt{\frac{a-b^\dagger}{a^\dagger-b}}\sqrt{a^\dagger a - b^\dagger b}$ in equation (4) denotes that the atom hopping from the lower state to the upper state is excited by the net number difference of two modes of photon field; in this process the interaction is proportional to $\sqrt{a^\dagger a - b^\dagger b}$, the square root of the net variation of the field intensity, while the term $\sqrt{a^\dagger a - b^\dagger b}\sqrt{\frac{a^\dagger-b}{a-b^\dagger}}\sigma_-$ denotes the opposite process. Such an interaction may happen in the competition of processes corresponding to nonlinear gain and nonlinear absorption in a two-photon medium. In order to solve this model, instead of using the ordinary two-mode Fock state $|m, n\rangle$ to represent the photon field, we introduce the state $\|\mathcal{N}_d, m\rangle$ to describe the photon field. The reason is let $D = a^\dagger a - b^\dagger b$, from the commutative relation

$$\left[D, \sqrt{\frac{a^\dagger - b}{a - b^\dagger}}\right] = \sqrt{\frac{a^\dagger - b}{a - b^\dagger}} \tag{7}$$

we have

$$D\|\mathcal{N}_d, m\rangle = \left[D, \left(\sqrt{\frac{a^\dagger - b}{a - b^\dagger}}\right)^{\mathcal{N}_d}\right]|m, m\rangle = \mathcal{N}_d\|\mathcal{N}_d, m\rangle \tag{8}$$

i.e. that $\|\mathcal{N}_d, m\rangle$ is the eigenstate of the photon-number difference D. The operators $e^{-i\Theta}$ and $e^{i\Theta}$ play the role of ascending and descending the net photon number, respectively,

$$\sqrt{\frac{a^\dagger - b}{a - b^\dagger}}\|\mathcal{N}_d, m\rangle = \|\mathcal{N}_d + 1, m\rangle \tag{9}$$

and

$$\sqrt{\frac{a - b^\dagger}{a^\dagger - b}}\|\mathcal{N}_d, m\rangle = \|\mathcal{N}_d - 1, m\rangle. \tag{10}$$

Moreover, one can prove that $\|\mathcal{N}_d, m\rangle$ also spans a complete set,

$$\sum_{\mathcal{N}_d=-\infty}^{\infty}\sum_{m=0}^{\infty}\|\mathcal{N}_d, m\rangle\langle\mathcal{N}_d, m\| = 1 \tag{11}$$

and has the orthogonality relations

$$\langle\mathcal{N}'_d, m'\|\mathcal{N}_d, m\rangle = \delta_{\mathcal{N}_d,\mathcal{N}'_d}\delta_{m,m'}. \tag{12}$$

Hence $\|\mathcal{N}_d, m\rangle$ is a good candidate for tackling with our new JCM. In the following we shall first calculate the time evolution of the energy eigenstate $|\Psi(t)\rangle$. Then we will calculate some concrete time-evolution quantities reflecting quantum aspects of this model, they are: the inversion $W(t)$ (see its definition in (24)), which accounts for certain collapse and revival phenomena, average phase and average net number difference.

2. The solution of the Schrödinger equation

Often it is convenient to solve the time-dependent dynamic problems in the interaction picture [8, 9]. Rewriting the Hamiltonian as

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 \tag{13}$$

where

$$\begin{aligned}\mathcal{H}_0 &= \varpi(a^\dagger a - b^\dagger b) + \frac{1}{2}\Omega\sigma_z\\ \mathcal{H}_1 &= \lambda\left[\sigma_+\sqrt{\frac{a-b^\dagger}{a^\dagger-b}}\sqrt{a^\dagger a - b^\dagger b} + \sqrt{a^\dagger a - b^\dagger b}\sqrt{\frac{a^\dagger-b}{a-b^\dagger}}\sigma_-\right]\end{aligned} \tag{14}$$

then using the Baker–Hausdorff formula

$$\mathrm{e}^{\alpha A}B\,\mathrm{e}^{-\alpha A} = B + \alpha[A, B] + \frac{\alpha^2}{2!}[A, [A, B]] + \cdots \tag{15}$$

and the commutative relation (7), we obtain

$$\mathcal{V} = \mathrm{e}^{\mathrm{i}\mathcal{H}_0 t}\mathcal{H}_1\,\mathrm{e}^{-\mathrm{i}\mathcal{H}_0 t} = \lambda\left[\sigma_+\sqrt{\frac{a-b^\dagger}{a^\dagger-b}}\sqrt{a^\dagger a - b^\dagger b}\,\mathrm{e}^{\mathrm{i}\Delta t} + \sqrt{a^\dagger a - b^\dagger b}\sqrt{\frac{a^\dagger-b}{a-b^\dagger}}\sigma_-\,\mathrm{e}^{-\mathrm{i}\Delta}\right] \tag{16}$$

where $\Delta = \Omega - \varpi$. In the interaction picture the equation of motion is

$$\mathrm{i}\frac{\partial}{\partial t}|\Psi(t)\rangle = \mathcal{V}|\Psi(t)\rangle. \tag{17}$$

From (9), (10) and (16) we see that the eigenstates of this JCM are always a linear combination of $\|\mathcal{N}_d, m\rangle|{+1}\rangle$ and $\|\mathcal{N}_d, m\rangle|{-1}\rangle$. And since the interaction energy can only cause transitions between the states $\|\mathcal{N}_d, m\rangle|{+1}\rangle$ and $\|\mathcal{N}_d + 1, m\rangle|{-1}\rangle$, $|\Psi(t)\rangle$ can be expressed as

$$|\Psi(t)\rangle = \sum_{\mathcal{N}_d=-\infty}^{\infty}\left[C_{+1,\mathcal{N}_d}(t)\|\mathcal{N}_d, m\rangle|{+1}\rangle + C_{-1,\mathcal{N}_d+1}(t)\|\mathcal{N}_d + 1, m\rangle|{-1}\rangle\right] \tag{18}$$

where $C_{+1,\mathcal{N}_d}(t)$ and $C_{-1,\mathcal{N}_d+1}(t)$ are the varying probability amplitudes, respectively. $\left|C_{+1,\mathcal{N}_d}(t)\right|^2$ represents the probabilities that at time t the atom is in the excited state and the two optical fields have a net photon-number difference $\mathcal{N}_d$, i.e. the system is in the state

$\|\mathcal{N}_d, m\rangle|+1\rangle$. A similar explanation can be given for $C_{-1,\mathcal{N}_d+1}$. Substituting equation (18) into equation (17), we have

$$\mathrm{i}\sum_{\mathcal{N}_d=-\infty}^{\infty}\left[\frac{\partial C_{+1,\mathcal{N}_d}(t)}{\partial t}\|\mathcal{N}_d, m\rangle|+1\rangle + \frac{\partial C_{-1,\mathcal{N}_d+1}(t)}{\partial t}\|\mathcal{N}_d+1, m\rangle|-1\rangle\right]$$
$$= \lambda\sum_{\mathcal{N}_d=-\infty}^{\infty}\left[\mathrm{e}^{\mathrm{i}\Delta t}C_{-1,\mathcal{N}_d+1}(t)\sqrt{\mathcal{N}_d+1}\|\mathcal{N}_d, m\rangle|+1\rangle + \mathrm{e}^{-\mathrm{i}\Delta}C_{+1,\mathcal{N}_d}(t)\sqrt{\mathcal{N}_d+1}\|\mathcal{N}_d+1, m\rangle|-1\rangle\right]. \tag{19}$$

Multiplying both the sides of equation (19) by $\langle -1|\langle\mathcal{N}_d+1, m\|$ and $\langle +1|\langle\mathcal{N}_d, m\|$ from the left, we obtain

$$\begin{aligned}\frac{\partial C_{-1,\mathcal{N}_{d+1}}(t)}{\partial t} &= -\mathrm{i}\lambda\sqrt{\mathcal{N}_d+1}\,\mathrm{e}^{-\mathrm{i}\Delta t}C_{+1,\mathcal{N}_d}(t)\\ \frac{\partial C_{+1,\mathcal{N}_d}(t)}{\partial t} &= -\mathrm{i}\lambda\sqrt{\mathcal{N}_d+1}\,\mathrm{e}^{\mathrm{i}\Delta t}C_{-1,\mathcal{N}_d+1}(t).\end{aligned} \tag{20}$$

The two coupled equations can be solved exactly subject to certain initial conditions,

$$\begin{aligned}C_{-1,\mathcal{N}_{d+1}}(t) &= \left\{C_{-1,\mathcal{N}_{d+1}}(0)\left[\cos\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right) + \frac{\mathrm{i}\Delta}{\Omega_{\mathcal{N}_d}}\sin\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\right]\right.\\ &\quad\left. - \frac{2\mathrm{i}\lambda\sqrt{\mathcal{N}_d+1}}{\Omega_{\mathcal{N}_d}}C_{+1,\mathcal{N}_d}(0)\sin\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\right\}\mathrm{e}^{-\mathrm{i}\Delta t}\\ C_{+1,\mathcal{N}_d}(t) &= \left\{C_{+1,\mathcal{N}_d}(0)\left[\cos\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right) - \frac{\mathrm{i}\Delta}{\Omega_{\mathcal{N}_d}}\sin\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\right]\right.\\ &\quad\left. - \frac{2\mathrm{i}\lambda\sqrt{\mathcal{N}_d+1}}{\Omega_{\mathcal{N}_d}}C_{-1,\mathcal{N}_{d+1}}(0)\sin\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\right\}\mathrm{e}^{\mathrm{i}\Delta t}\end{aligned} \tag{21}$$

where

$$\Omega^2_{\mathcal{N}_d} = \Delta^2 + 4\lambda^2(\mathcal{N}_d+1). \tag{22}$$

If initially the atom is in the excited state $|+1\rangle$, then $C_{+1,\mathcal{N}_d}(0) = C_{\mathcal{N}_d}(0)$ and $C_{-1,\mathcal{N}_{d+1}}(0) = 0$, here $C_{\mathcal{N}_d}(0)$ is the probability amplitude for the photon-number difference of the fields alone, we then obtain

$$\begin{aligned}C_{-1,\mathcal{N}_{d+1}}(t) &= -C_{\mathcal{N}_d}(0)\frac{2\mathrm{i}\lambda\sqrt{\mathcal{N}_d+1}}{\Omega_{\mathcal{N}_d}}\sin\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\mathrm{e}^{-\mathrm{i}\Delta t}\\ C_{+1,\mathcal{N}_d}(t) &= C_{\mathcal{N}_d}(0)\left[\cos\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right) - \frac{\mathrm{i}\Delta}{\Omega_{\mathcal{N}_d}}\sin\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\right]\mathrm{e}^{\mathrm{i}\Delta t}.\end{aligned} \tag{23}$$

$C_{-1,\mathcal{N}_{d+1}}(t)$ and $C_{+1,\mathcal{N}_d}(t)$ describe the time evolution completely, and all the remarkable time-dependent quantities of the JCM can be derived from them.

3. The inversion $W(t)$, the time evolution of phase and number difference

The inversion $W(t)$ of an atom-field system is one important quantity which exhibits pure quantum effects, which is related to the probability amplitude $C_{-1,\mathcal{N}_{d+1}}(t)$ and $C_{+1,\mathcal{N}_d}(t)$

by the expression,

$$
\begin{aligned}
W(t) &= \sum_{\mathcal{N}_d=-\infty}^{\infty} \left[\left| C_{+1,\mathcal{N}_d}(t) \right|^2 - \left| C_{-1,\mathcal{N}_d}(t) \right|^2 \right] \\
&= \sum_{\mathcal{N}_d=-\infty}^{\infty} \rho_{\mathcal{N}_d}(0) \left[\frac{\Delta^2}{\Omega_{\mathcal{N}_d}^2} + \frac{4\lambda^2(\mathcal{N}_d+1)}{\Omega_{\mathcal{N}_d}^2} \cos\left(\Omega_{\mathcal{N}_d} t\right) \right] \qquad (24)
\end{aligned}
$$

where we have used equation (23). In equation (24), $\rho_{\mathcal{N}_d}(0) = \left| C_{\mathcal{N}_d}(0) \right|^2$ is the initial probability that the net photon-number difference of the two optical modes is $\mathcal{N}_d$, and it determines the relative weight for each value of $\mathcal{N}_d$ in the summation. Because the Rabi oscillations in summation with different $\mathcal{N}_d$ have different frequencies, as time increases they become uncorrelated and interfere destructively, this therefore leads to a collapse of inversion. Moreover, the discrete nature of the net photon-number difference distribution leads to a partial rephasing or revival of such oscillations.

From equation (23) we can also calculate

$$
\begin{aligned}
\langle \Psi(t) | \, \mathrm{e}^{-\mathrm{i}\Theta} | \Psi(t) \rangle &= \sum_{\mathcal{N}_d=-\infty}^{\infty} \left[C_{+1,\mathcal{N}_d'}^*(t) \langle +1 | \langle \mathcal{N}_d', m \| + C_{-1,\mathcal{N}_d'+1}^*(t) \langle -1 | \langle \mathcal{N}_d'+1, m \| \right] \\
&\quad \times \sum_{\mathcal{N}_d=-\infty}^{\infty} \left[C_{+1,\mathcal{N}_d}(t) \| \mathcal{N}_d+1, m \rangle | +1 \rangle + C_{-1,\mathcal{N}_d+1}(t) \| \mathcal{N}_d+2, m \rangle | -1 \rangle \right] \\
&= \sum_{\mathcal{N}_d=-\infty}^{\infty} \left[C_{+1,\mathcal{N}_d}^*(t) C_{+1,\mathcal{N}_d-1}(t) + C_{-1,\mathcal{N}_d+1}^*(t) C_{-1,\mathcal{N}_d}(t) \right] \qquad (25)
\end{aligned}
$$

and

$$
\langle \Psi(t) | \, \mathrm{e}^{\mathrm{i}\Theta} | \Psi(t) \rangle = \sum_{\mathcal{N}_d=-\infty}^{\infty} \left[C_{+1,\mathcal{N}_d}^*(t) C_{+1,\mathcal{N}_d+1}(t) + C_{-1,\mathcal{N}_d+1}^*(t) C_{-1,\mathcal{N}_d+2}(t) \right] \qquad (26)
$$

then,

$$
\begin{aligned}
\langle \Psi(t) | \cos\Theta | \Psi(t) \rangle &= \frac{1}{2} \sum_{\mathcal{N}_d=-\infty}^{\infty} \left[C_{+1,\mathcal{N}_d}^*(t) C_{+1,\mathcal{N}_d-1}(t) + C_{-1,\mathcal{N}_d+1}^*(t) C_{-1,\mathcal{N}_d}(t) \right. \\
&\quad \left. + C_{+1,\mathcal{N}_d}^*(t) C_{+1,\mathcal{N}_d+1}(t) + C_{-1,\mathcal{N}_d+1}^*(t) C_{-1,\mathcal{N}_d+2}(t) \right] \\
&= \sum_{\mathcal{N}_d=-\infty}^{\infty} \mathrm{Re}\left[C_{+1,\mathcal{N}_d}^*(t) C_{+1,\mathcal{N}_d-1}(t) + C_{-1,\mathcal{N}_d}^*(t) C_{-1,\mathcal{N}_d-1}(t) \right]. \qquad (27)
\end{aligned}
$$

Substituting equation (23) into equation (27), we obtain

$$
\begin{aligned}
\langle \Psi(t) | \cos\Theta | \Psi(t) \rangle &= \sum_{\mathcal{N}_d=-\infty}^{\infty} \mathrm{Re} \left\{ C_{\mathcal{N}_d}^*(0) C_{\mathcal{N}_d-1}(0) \left[\cos\left(\frac{\Omega_{\mathcal{N}_d} t}{2}\right) \cos\left(\frac{\Omega_{\mathcal{N}_d-1} t}{2}\right) \right. \right. \\
&\quad + \left(\frac{\Delta^2 + 4\lambda^2 \sqrt{\mathcal{N}_d+1}\sqrt{\mathcal{N}_d}}{\Omega_{\mathcal{N}_d} \Omega_{\mathcal{N}_d-1}} \right) \sin\left(\frac{\Omega_{\mathcal{N}_d} t}{2}\right) \sin\left(\frac{\Omega_{\mathcal{N}_d-1} t}{2}\right) \\
&\quad \left. \left. + \frac{\mathrm{i}\Delta}{\Omega_{\mathcal{N}_d}} \sin\left(\frac{\Omega_{\mathcal{N}_d} t}{2}\right) \cos\left(\frac{\Omega_{\mathcal{N}_d-1} t}{2}\right) - \frac{\mathrm{i}\Delta}{\Omega_{\mathcal{N}_d-1}} \sin\left(\frac{\Omega_{\mathcal{N}_d-1} t}{2}\right) \cos\left(\frac{\Omega_{\mathcal{N}_d} t}{2}\right) \right] \right\}. \qquad (28)
\end{aligned}
$$

In particular, when $\Delta = \Omega - \omega = 0$, equation (28) becomes

$$\langle\Psi(t)|\cos\Theta|\Psi(t)\rangle = \sum_{\mathcal{N}_d=-\infty}^{\infty} \mathrm{Re}\left\{C^*_{\mathcal{N}_d}(0)C_{\mathcal{N}_d-1}(0)\left[\cos\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\cos\left(\frac{\Omega_{\mathcal{N}_d-1}t}{2}\right)\right.\right.$$
$$\left.\left. + \sin\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\sin\left(\frac{\Omega_{\mathcal{N}_d-1}t}{2}\right)\right]\right\}$$
$$= \sum_{\mathcal{N}_d=-\infty}^{\infty} \mathrm{Re}\left[C^*_{\mathcal{N}_d}(0)C_{\mathcal{N}_d-1}(0)\cos\left(\frac{\Omega_{\mathcal{N}_d}-\Omega_{\mathcal{N}_d-1}}{2}t\right)\right]$$
$$= \sum_{\mathcal{N}_d=-\infty}^{\infty} \mathrm{Re}\left[C^*_{\mathcal{N}_d}(0)C_{\mathcal{N}_d-1}(0)\cos\left(\frac{\lambda}{\sqrt{\mathcal{N}_d+1}+\sqrt{\mathcal{N}_d}}t\right)\right] \tag{29}$$

which manifestly exhibits the cosine evolution of time. Physically, without loss of generality, we can always assume $\mathcal{N}_d \geqslant 0$, which means that the photon numbers in one mode are always larger than in the other mode; from equation (29) we can immediately see that the time evolution of average phase is determined partly by the initial photon-number difference probability amplitude. While the expectation value of the number-difference operator in the state $|\Psi(t)\rangle$ is

$$\langle\Psi(t)|D|\Psi(t)\rangle = \sum_{\mathcal{N}_d=-\infty}^{\infty}\left[C^*_{-1,\mathcal{N}'_d+1}(t)\langle -1|\langle\mathcal{N}'_d+1, m\| + C^*_{+1,\mathcal{N}'_d}(t)\langle +1|\langle\mathcal{N}'_d, m\|\right]$$
$$\times D\sum_{\mathcal{N}_d=-\infty}^{\infty}\left[C_{+1,\mathcal{N}_d}(t)|\mathcal{N}_d, m\rangle|+1\rangle + C_{-1,\mathcal{N}_d+1}(t)|\mathcal{N}_d+1, m\rangle|-1\rangle\right]$$
$$= \sum_{\mathcal{N}_d=-\infty}^{\infty}\left[\mathcal{N}_d|C_{+1,\mathcal{N}_d}(t)|^2 + (\mathcal{N}_d+1)|C_{-1,\mathcal{N}_d+1}(t)|^2\right]. \tag{30}$$

Substituting equation (23) into equation (30) we have

$$\langle\Psi(t)|D|\Psi(t)\rangle = \sum_{\mathcal{N}_d=-\infty}^{\infty}\left\{\mathcal{N}_d\left|C_{\mathcal{N}_d}(0)\right|^2\left[\cos^2\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\right.\right.$$
$$\left.\left. + \frac{1}{\Omega^2_{\mathcal{N}_d}}\left(\Delta^2 + \frac{4\lambda^2(\mathcal{N}_d+1)^2}{\mathcal{N}_d}\right)\sin^2\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\right]\right\}. \tag{31}$$

When $\Delta = \Omega - \omega = 0$,

$$\langle\Psi(t)|D|\Psi(t)\rangle = \sum_{\mathcal{N}_d=-\infty}^{\infty}\left|C_{\mathcal{N}_d}(0)\right|^2\left[\mathcal{N}_d\cos^2\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right) + \sin^2\left(\frac{\Omega_{\mathcal{N}_d}t}{2}\right)\right]. \tag{32}$$

In summary we have applied the newly derived number-operational phase entangled state to solving a number-phase entangled Jaynes–Cummings model. The time evolution of the phase and number difference is calculated. The model also exhibits collapse-revival phenomena.

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